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THE COMPLEXITY PROPERTIES OF PROBABILISTIC AUTOMATA WITH ISOLATED CUT POINT

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Abstract. A probabilistic automaton (PA) which accepts a language with e-isolated cut point $\frac{1}{2}$ corresponds to a PA which computes with $(\frac{1}{2}-e)$ bounded error probability. Let $P(L, e)$ be the minimal number of states of a PA necessary for accepting a language L with e-isolated cut point $\frac{1}{2}$. It is shown that there are languages L^k, $1<k<\infty$ and an infinite sequence of numbers $0<e_1<e_2<\cdots<\frac{1}{2}$ such that for all $i\geq 1$, $P(L^k, e_i)/P(L^k, e_{i+1})\to 0$ when $k\to\infty$. It is also shown that the probabilistic recognition of the language W^k is more effective than that of the L^k.

1. Introduction

The notion of probabilistic automata, or shortly PA, was introduced by Rabin [4]. A PA which accepts a language with e-isolated cut point $\frac{1}{2}$ corresponds to a PA which computes with $(\frac{1}{2}-e)$ bounded error probability. Rabin [4] proved that PAs with isolated cut point can accept only regular languages, i.e., can do no more than deterministic automata (shortly DA). But as PAs compute with some error probability, it is expected that PAs require a smaller number of states than any DA recognizing the same language.

For several languages there exist PAs recognizing these languages with an isolated cut point and these PAs require a smaller number of states than any DA recognizing these languages [1, 4]. The examples of languages for which PAs do not have such advantages were given in [3].

From [3] it also follows that there are languages with the following properties: the number of states of PAs recognizing them does not depend on the value of isolation of the cut point.

In this paper we present a sequence of languages. For any PA recognizing these languages, the number of states strongly depends on the degree of isolation of the cut point. In addition we demonstrate two examples of languages with the following properties:

(i) the deterministic complexity of these languages (the number of states of the minimal automaton accepting them) is "nearly" the same;

(ii) the probabilistic complexity of these languages is rather different.

2. Definitions and results

The set of all words, including the empty word over the alphabet $X=\{0,1\}$ is denoted by X^*. Subsets of X^* are referred to as languages over X. The length of

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